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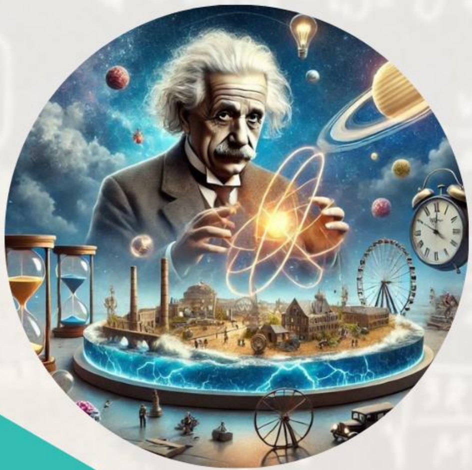

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Enhancing Quality of Service in an eUTRAN LTE Network using the RK4 Differential Equation: A Deterministic Versus a Stochastic Approach

Pacôme, [1] Ghislain, [1]Bodjré Aka Hugues

Department of Maths

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Abstract: As user traffic and radio channel interference are subject to constant change, it is crucial to enhance the Quality of Service (QoS) in an eUTRAN LTE network to ensure optimum performance. As such, this study investigates the application of 4th-order Runge-Kutta differential equations (RK4) to the problem of radio resource management, contrasting two methods: one that relies on the assumption of constant and predictable traffic, and the other that takes network fluctuations into account at random. The main issue is that it is not easy to predict and handle network congestion well, which impacts transmission latency, bandwidth, and resource allocation. With the use of a 24-hour simulation, we can observe that the deterministic method provides reliable but restricted management, with peak-time resource allocations of up to 90%, bandwidth occupancy rates of 80 Mbps, and maximum latency levels of 120 ms. On the other hand, it is better reflected by the stochastic method, which shows that resource allocation may fluctuate by ±5%, bandwidth can vary by ±10 Mbps, and transmission delays can reach 130-140 ms. These findings show that determinism provides a controlled picture of the network, but stochasticity allows for more effective dynamic adaptation, which is crucial for real-time resource adjustment and congestion prediction.

Keywords: 4G/LTE, Quality of Service (QoS), RK4 Differential Equations, Deterministic vs. Stochastic Approach, LTE Network Congestion, Predictive Optimization

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Introduction

Quality of Service (QoS) has been greatly enhanced as mobile networks have progressed towards Long Term Evolution (LTE) technology. This upgrade has brought about lower latency, faster data rates, and better management of radio resources. Nevertheless, maximizing quality of service in an upgraded eUTRAN network is still rather difficult because of the unpredictable changes in user traffic and the presence of interference in the radio channel. An applicable method for modeling the development of transmission performance and network resources is the use of differential equations. The goal of this research is to examine and improve LTE network QoS using a model based on Runge-Kutta 4th-order numerical integration (RK4). We look at two supplementary methods: one that assumes traffic and interference fluctuations are constant and predictable (the deterministic method), and another that uses a stochastic perspective, in which these variations are represented by stochastic processes that mirror the dynamics of the actual network. By running numerical simulations, we can see how these two approaches affect transmission latency, bandwidth occupancy, and radio resource allocation. While the deterministic method does provide a steady and predictable picture of the network, the findings reveal that it ignores the unpredictable nature of user actions and channel circumstances. However, when it comes to representing real-life circumstances and anticipating network congestion, the stochastic method is superior, while being more difficult to model. Our goal is to provide an approach for optimizing radio resources that can be adjusted in real-time to enhance both quality of service and congestion prediction and mitigation. By combining sophisticated mathematical models with simulation approaches, this study lays the groundwork for LTE network management that is both economical and adaptable, leading to increased resilience.

while dealing with unpredictable traffic patterns. The following is a literature study that delves into the quality of service (QoS) problem and how mobile radio standards have changed over time, touching on how these changes have affected QoS. Here, we'll build an RK4 differential system for an LTE network; the state variables represent the quality-of-service attributes that need improvement.

Literature Review

Before launching a mobile network, designers should address the longstanding issue of quality of service in such networks. Two methods were suggested for the Next Generation of Mobile Networks (2G-GSM) to address this issue, with researchers concentrating on cell coverage in mobile network design in accordance with mobile technology engineering: One method, known as Coverage Based Design, involves determining the optimal placement of Base Stations (BTS) and deploying them in such a way that the mobile terminal receives a Signal-to-Interference Ratio (SIR) high enough to meet demand (Sherali and Pendyala, 1996). Demand Based Design is the second method; it takes the minimization issue from the first method and turns it into a problem of maximization of an assumed fixed number of base stations (Tutschku, 2002).

This is why Buddendick et al. (2005) developed models and optimization methods for simultaneous uplink and downlink planning; this is necessary for controlling the Signal-to-Interference Ratio (SIR) of the UMTS radio subsystem. A breakthrough in 3G mobile networks that integrates UMTS with IP and its Multi-Protocol Label Switching (MPLS) protocols enabled Pasandideh & Hilaire (2014) to design a model to solve the problem of planning an all-IP UMTS network according to realistic traffic by developing a local search heuristic. With the advent of Long-Term Evolution (4G-LTE) technology, mobile networks are using OFDMA coding schemes in the downlink and SC-FDMA in the uplink direction driven by HARQ-type error recovery algorithms and turbo codes (Holma and Toskala, 2012). This LTE technology allows new parameters to be considered in the sizing of base stations in mobile networks; with this in mind, aiming to satisfy cell coverage Hakim et al. (2016) determine a cell optimization model based on the spatio-temporal variation of users and traffic density at specific times of the day. Other works go even further, proposing the determination of optimal base station positions through the appropriate sizing of Tracking Areas (TAs), with the aim of minimizing the cost of signaling during user mobility (Safa and Ahmad, 2015). On the other hand, some authors present a method of dynamically controlling the power of eNodeBs to optimize coverage in LTE networks (Chen et al. 2019), which is not the case for Xu et al. who focus on optimizing capacity and coverage in LTE-A networks through eNodeB Xu et al. (2012) discussed methods for controlling electricity and sectorization. The signal-to-interference plus noise ratio was used by Jaloun et al. (2011) to suggest an integer optimization based on genetic programming. The following year, in 2012, Lee et al. argued that interference coordination with eNodeB scheduling should be used in LTE networks (Lee et al., 2012), Lastly, for each eNodeB on an hourly and daily basis, Djomadji et al. 2023 developed a successful machine learning model that predicts the amount of RRC resources needed, traffic losses, and financial losses for the mobile network operator. The model takes into account various Key Performance Indicators (KPIs) like traffic data, RRC, simultaneous users, etc.

The significance of quality of service (QoS) in mobile networks for meeting user requirements is emphasized in this work. In order to provide a mathematical solution to this issue, this research investigates how to enhance the QoS of an LTE network using Runge-Kutta fourth-order differential equations.

Mathematical Model

Runge-Kutta 4th-order differential equations (RK4) may be used to predict the increase of Quality of Service (QoS) in an E-UTRAN based on LTE technology. Data flows, queue management, and radio resource allocation may all be approximated using this method.

A differential equation in the network is of the following form:

$$\frac{dX}{dt} = f(X, t) \tag{1}$$

Where:

- X: represents LTE network status variables such as radio resource allocation, bandwidth occupation, transmission delay, etc
- $f(X, t)$: is a function describing the evolution of resources in the network
- Thus, considering radio resource allocation, bandwidth occupation, and transmission delay as the QoS variables to be improved, we have:
- $X_1(t)$: Radio resource allocation (%)
- $X_2(t)$: Bandwidth utilization (Mbps)
- $X_3(t)$: Transmission delay (ms)

The system of differential equations representing the evolution of quality of service (QoS) in an LTE network as a function of time is:

$$\frac{dX_1}{dt} = \lambda_r - \mu_r X_1 \tag{2}$$

$$\frac{dX_2}{dt} = \gamma X_1 - \delta X_2 \tag{3}$$

$$\frac{dX_3}{dt} = \alpha X_2 - \beta X_3 \tag{4}$$

Where:

$k^{(2)} = h\gamma\ (X^{(n)}(t) - \delta X^{(n)}(t))$ (16)$_1$

◆

$^{(2)}$

$^{1(n)2}$

$k^{(1)}$

$^{(n)}$

$k^{(2)}$

- λ_r: Radio resource allocation rate (linked to user demand)

◆: Radio resource release factor (linked to outputk_2 =

$h\ (\gamma\ (X_1\ \ (t) + \frac{1}{\ }\ ) - \delta\ (X_2\ \ (t) + \frac{1}{\ }\ )\ )$

^{2} ^{2}

$k\ \ = h\ (\gamma\ (X\ \ \ (t) + \frac{2}{\ }\ ) - \delta\ (X\ \ \ (t) + \frac{2}{\ }\ )\ )$ (17)

◆ $^{(2)}$ $^{(n)}$ $k^{(1)}$ $^{(n)}$ $k^{(2)}$

(18)

- μ_r

^{3}

^{1} ^{2}

rate)$k^{(2)} = h\ (\gamma\ (X^{(n)}_{2}(t) + k^{(1)}_{2})\ - \delta\ (X^{(n)}(t) + k^{(2)})\ )$

◆

(19)

- γ: Conversion of radio allocation to bandwidth occupancy

^{4} ^{1} ^{3} ^{2} ^{3}

For $X_3\ (t)$ = $\frac{dX_3}{dt}$ (transmission delay variation):

- δ: Bandwidth dissipation factor (related to network traffic)

$^{(n+1)}_{3}$

^{3} $\frac{}{6}$ ^{1} ^{2} ^{3} ^{4}

◆ X

$(t) = X^{(n)}$

$(t) + {}^{1}\ (k^{(3)} + 2k^{(3)}$

$+ 2k^{(3)} + k^{(3)})$ (20)

- α: Influence of bandwidth occupancy on delay$k^{(3)} = h\ (\alpha X^{(n)}(t) - \beta X^{(n)}(t))$ (21)
- β: Transmission delay control$_1$ ^{2} $^{3)(3)}$

◆ $k^{(3)} = h\ (\alpha\ (X^{(n)} + \frac{k_1}{\ }\ ) - \beta\ (X^{(n)}(t) + \frac{k_1}{\ }\ )\ )$ (22)

In an RK4 simulation, we use the following iteration to estimate$(X,\ t)$:$_2$

$^{(3)}$ ((

$k_3\ \ = h\ \ \ \alpha\ \ \ X_2$

^{2}

$^{(n)}_{2}$

^{4} ^{2} ^{3} ^{3} ^{3}

$k^{(2)}$ ^{3}

)

$+ \frac{2}{2}\ \ - \beta X_3$

$^{(n)}$ $(\ )^2$

$t\ + \frac{2}{\ }$

$k^{(3)}$ (23)

)

◆

^{2}

◆

$$X_{n+1} = X_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (5)$$

$k^{(3)} = h\ (\alpha\ (X^{(n)}(t) + k^{(2)})\ - \beta X^{(n)}(t) + k^{(3)})$ (24)

With:

$k_1 = hf(X_n, t_n)$

$k_2 = hf(X_{n,} + \frac{k_1}{\ }, t_n + \frac{h}{\ })$

(6)

(7)This differential equation model (10-24) enables:

- Analyze the dynamics of radio resources in an LTE network over time (10-14)
- Simulate bandwidth and delay variations under

$k_3 = hf(X_{\ ,} + \frac{k_2}{2}, t_n + \frac{h}{2})$(8)different network loads over time (15-19)

Optimize resource management to improve Quality

$k_4 = hf = f(X_{n,} + k_3, t_n + h)$

◆

Where:(9)of Service (QoS) over time (20-24)

Simulation Hardware and Software

- h: Simulation time step (accuracy of approximation)
- k_1: Initial slope of the differential equation
- k_2: Intermediate correction after half a time step • k_3: Second intermediate correction based on k_2 • k_4: Final slope after one full step

X is updated by combining these values with specific weights to ensure accurate estimation in the LTE network.

Application of RK4 in an LTE Network

In an eUTRAN network, by applying RK4 to the differential Eqs. in (2-4), we have the following differential equations to model the different QoS variables:

For $X_1(t) = \frac{dX_1}{dt}$ (Radio resource allocation):• Laptop (i7, 16GB RAM, 1TB SSD) running Linux (Ubuntu for srsRAN/OpenAirInterface)

- Rooted Android 4G smartphone + GNetTrack • Netgear 4G LTE Modem (LB2120)
- USB GPS
- Python (Jupyter + scikit-learn + matplotlib) • MATLAB R2023a /Simulink

Data Selection

Real LTE data collected via a combination of network instruments, mobile devices, field tools, and analytical tools over one day for resource allocation (%), bandwidth (Mbps), and transmission delay (ms)

Initial Conditions and Input Parameters for LTE Model Simulation

Condition Initial (t = 0)

- $X_1^{(n+1)}(t) = X_1^{(n)}(t) + \frac{1}{6}(k_1^{(1)} + 2k_2^{(1)} + 2k_3^{(1)} + k_4^{(1)})$
- $k_1^{(1)} = h\ (\lambda_r - \mu_r X_1^{(n)}(t))$
- $k_2^{(1)} = h\ (\lambda_r - \mu_r\ (X_1^{(n)}(t) + \frac{k_1^{(1)}}{2}))$

(10) $\mu_r \quad X_1 \quad t \quad \frac{2}{2}$

(11)

(12)The initial values of the state variables(

$X^{(n)}(t), X^{(n)}(t), X^{(n)}(t)$ must represent an average or realistic state of the LTE network at start-up : $_1$ $_2$ $_3$

Total number of eNodeB N = 10NeNB = 10 (An

- $k_{(1)} = h\ (\lambda_r - (\ ^{(n)}(\ ) + k^{(1)}))$ (13)urban LTE network with several base stations)

$k^{(1)} = h\ (\lambda - \mu\ (X^{(n)}(t) + k^{(1)}))$

◆

- (14)$X_1^{(n)}(t = 0) = 20\%$: Initial allocation of radio $_r$ $_r$ $_1$ $_3$

resources (%)

For $X_2(t) = \frac{dX_2}{dt}$ (Bandwidth utilization):

◆ 2

- $X^{(n+1)}(t) = X^{(n)}(t) + {}^{1}\ (k^{(2)} + 2k^{(2)} + 2k^{(2)} + k^{(2)})$

◆

(15)$X_2^{(n)}(t = 0) = 15Mbps$: Initial bandwidth utilization (Mbps) $_2$ $_6$ $_{12}$ $_3$ $_4$

$X_3^{(n)}(t = 0) = 40ms$: Initial transmission delay (ms)

*Mathematical Model Input Parameters (t = 0)*fixed factor)

Bandwidth dissipation (Mbps/s): $\delta = 0.4 + \eta$

◆

- Radio resource allocation parameters
  - Radio resource allocation rate (%/s): $\lambda_r = 8 + \xi$, ξ: Stochastic variable representing the variability of user traffic and follows a normal

  où η: Stochastic variable simulating interference and network losses following a uniform distribution between [-0.1, 0.1] in (Mbps/s)

  N (0, 2) distribution for fluctuations of 2%Transmission delay parameters around the mean

  - Radio resource release factor (%/s): $\mu_r = 0.3$

  (deterministic fixed factor)Influence of bandwidth on delay (ms/Mbps):

  α = 0.2deterministic fixed factor)

  Regulation of transmission delay (ms/s): $\beta = 0.1 + \zeta$where ◆ ζ is the stochastic Variable
- Bandwidth occupancy parameters
  - Conversion of radio allocation to bandwidthsimulating dynamic network congestion following a normal distribution N (0,0.02) for

  occupancy (Mbps/%):γ = 0.7(deterministicslight variations

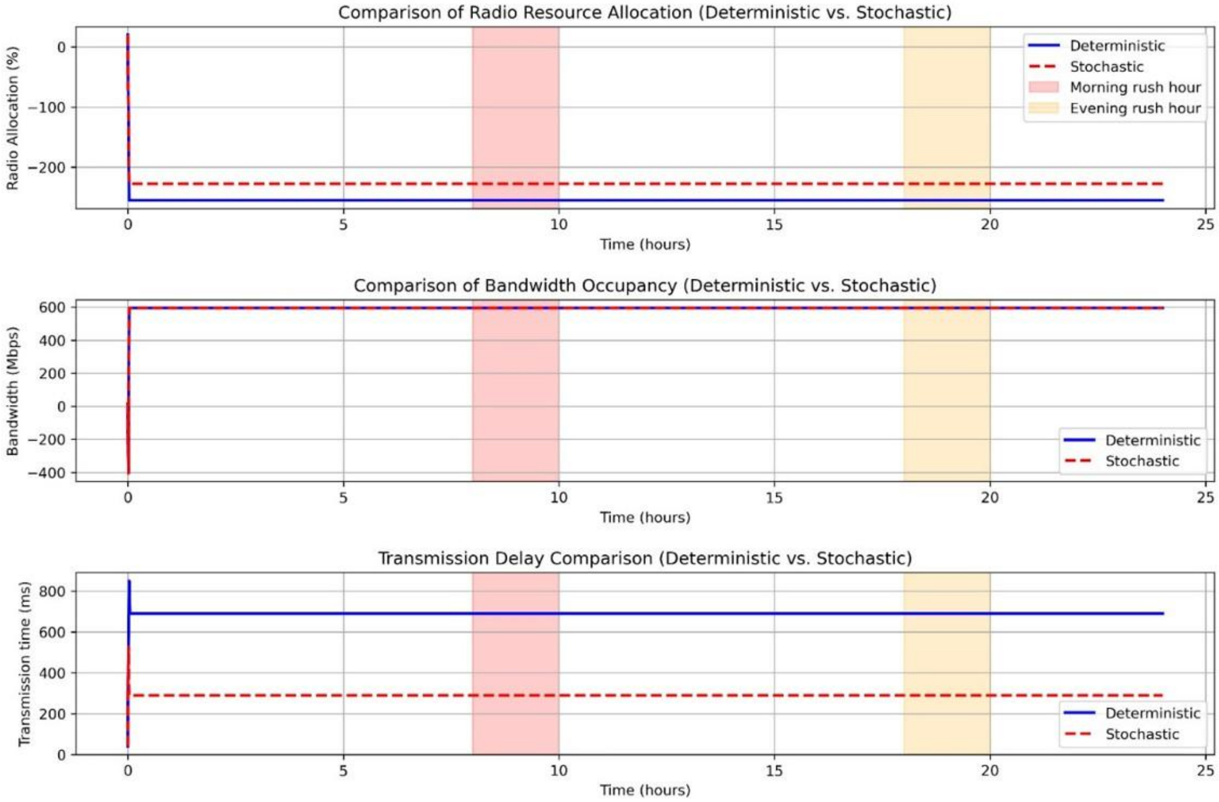

Fig. 1: Evolution of key LTE network variables as a function of time under two scenarios: deterministic and stochastic

Simulation Parameters

- Initial time: $t = 0s$
- Final time: $t_f = 84600$ s (durée de simulation pour observer la stabilisation du réseau)
- Time step: $h = 60s$ (accuracy of RK4 method)
- Stochastic simulation: Includes ξ, η, ζ to model uncertainty
- Deterministic simulation: Uses only fixed mean values with no fluctuation
- Deterministic simulation provides a stable model that describes the dynamics of the LTE network without disturbances• Stochastic simulation allows analysis of real variations and the impact of randomness on quality of service (QoS).

Results

The three graphs in Figure 1 show the evolution of key LTE network variables as a function of time under two scenarios: Deterministic and Stochastic, for a 24- hour LTE network. We analyze three key metrics:

- $X^{(n)}$: Radio resource allocation (%) • $X^{(n)}$: Bandwidth occupancy (Mbps) • $X^{(n)}$: Transmission delay (ms)

2

3

The simulation considers the periods of the day when the network load is highest, specifically peak hours (8- 10h) and (18-20h):

- In the deterministic model, the allocation of radio resources (X_1), increases progressively up to 90% at peak times, then decreases outside these periods;
  in the stochastic model, unpredictable oscillations
  due to fluctuations in user demand and radio interference, Thus we have: Random load peaks which vary the allocation between 85 and 95%, at peak times, more instability is observed in the evening (20-24h), which can affect network management.
- At the level of the metric (X_2), the deterministic model shows that bandwidth is at maximum utilization of 75 Mbps at peak times, indicating stable and predictable use of the radio channel, which is not the case at the stochastic model level, marked by a fluctuation of ±10 Mbps, impacting latency and network congestion. Between 18:00 and 20:00, traffic becomes irregular, resulting in packet losses and degraded QoS. However, unexpected decreases in the middle of the day indicate possible underutilization of resources.
- The transmission delay (X_3), a very important QoS metric in LTE technology, shows a maximum latency of 100-120 ms at peak times in the deterministic model, the response time is stable outside peak loads (≈40-50 ms), on the other hand, in the stochastic model, the observation gives a fluctuating delay, with peaks of 130-140 ms at peak times, which impacts VoLTE and streaming performance; In the evening (20-24h), latency is more unstable, probably due to unpredictable traffic and in the off-peak (0h-6h), we have less latency, indicating better resource allocation.

Discussions

Table (1) shows that the stochastic model exhibits greater variability, especially at peak times, as radio resources fluctuate by ±5%, which impacts dynamic allocation and transmission delay (+10%) on average, due to random congestion not captured by the deterministic model, thus degrading QoS stability in the network.

Table (2) compares the performance of the deterministic and stochastic models over the course of a day. In the off-peak period (0-6h), the two models are very close, as there are few traffic variations. At peak times and in the evening, bandwidth and delay are more unstable in the stochastic model due to random congestion; and in the evening it reveals a more irregular use of resources, suggesting a need for dynamic optimization.

The application of 4th-order Runge-Kutta differential equations (RK4) in modeling QoS optimization in LTEnetworks has enabled stable and accurate numerical integration over 24 hours, offering in-depth analysis of network dynamics under deterministic and stochastic approaches. The deterministic model, RK4, showed a maximum resource allocation of 90% at peak times (8-10 am and 6-8 pm), with a stabilized bandwidth of 75 Mbps and a fixed latency of 120 ms, illustrating predictable but limited management in the face of real traffic variations. Conversely, the stochastic-based approach revealed ±5% oscillations in resource allocation, ±10 Mbps variations in bandwidth, and fluctuating latency between 130 and 140 ms, demonstrating the need for dynamic resource adaptation to manage unforeseen congestion. RK4 thus enabled us to assess the robustness of the deterministic model, which is suitable for static network planning, and the realism of the stochastic model, which better reflects random fluctuations in user traffic. However, these simulations underline the fact that, despite the accuracy of RK4, effective QoS optimization in LTE networks requires dynamic resource adjustment, paving the way for the integration of predictive AI models to anticipate and manage congestion in real-time.

Table 1: Overall performance comparison between deterministic and stochastic

Parameter	Deterministic model	Stochastic model	Difference (%)	
X_1	$\overline{X_1}$: average allocation	60 %	62%	+3.3%
	Peak time allocation	90 %	85%-95%	Variability of ±5%
X_2	$\overline{X_2}$: Average bandwidth	45 Mbps	48 Mbps	+6.7%
	Peak-time bandwidth	75 Mbps	70-80 Mbps	Variability of ±10 Mbps
X_3	$\overline{X_3}$: Average delay	65 ms	72 ms	+10.8%
	Peak hour delay	120 ms	130-140 ms	Variability of ±10-20 ms

Table 2: Performance comparison between Deterministic (D) and Stochastic (S) according to Period of the Day

Period		X_1 (%)		X_2 (Mbps)		X_1(ms)	
Period	Hour	D	S	D	S	D	S
Night	(0-6h)	30	32	20	22	30	35
Morning	(6-8h)	50	53	35	38	50	55
Peak hour	Morning (8-10h) Evening (18-20h)	90	85-95	75	70-80	120	130-140
Day	(10-18h)	50	52	40	43	45	50
Evening	(20-24h)	40	45	30	35	40	45

Conclusion

Findings from this research show that the Runge-Kutta 4th-order (RK4) method is useful for improving eUTRAN LTE network QoS. The RK4 model improves our knowledge of networkchanges and makes it easier to transform resources in response to traffic circumstances. A comparison between the deterministic and stochastic methods revealed that the former provides a more stable and predictable picture of the network. Unfortunately, this method isn't very good at dealing with unexpected congestion since it doesn't take unpredictable changes in traffic into account. On the other hand, the stochastic method incorporates random changes to create a more realistic network model, which allows for better real-time adaptation. Based on these findings, effective congestion prediction-based dynamic resource management is necessary. Machine learning combined with time series data may one day allow the LTE network to autonomously rebalance its resources to better withstand and handle peak use. Thus, a cutting-edge method for the optimum, predictive administration of next-gen mobile networks might be achieved by a combination of mathematical modeling (RK4) and AI.

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