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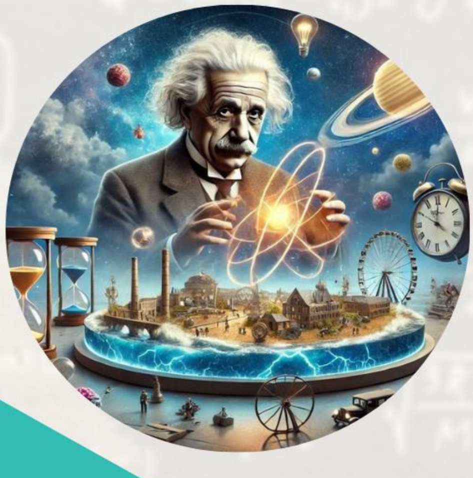

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Regarding Hyper d-Algebras

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Abstract: An assortment of hyper algebraic structures, such as hyper BCK-algebras, hyper BE-algebras, hyper UP-algebras, and hyper KU-algebras, were introduced subsequent to hyper algebra's inception. Hyper d-Algebra, an extension of d algebra, is introduced and associated characteristics are investigated in this research article. In addition, we cover Hyper d-Algebra, its sub-algebra, its ideal, and its Cartesian product.

Keywords: d-Algebra, Hyper d-Algebra, Hyper Sub Algebra of Hyper d-Algebra, Hyper Ideal

Introduction

According to Jun, hyperstructures have several practical and theoretical uses in mathematics (1999; 2001). References for hyper-structure theory and its mathematical and computational applications may be found in Torkzadeh and Zahedi (2006) and Dejen (2020). Jun and Zahedi (2000) studied some associated characteristics of hyper BCK-algebras, an extension of BCK-algebras, and applied hyperstructures to them.

Hyper BE algebra and related topics were covered by Akefe Radfar, Akbar Rezaei, and Arsham Borumand Saeid in their 2014 publication (Radfar et al., 2014).

Case 2. 2. Envision X as a set consisting of the elements 0, a, b, and c. Afterwards, the procedure
* on X is defined by Table 1.

Clearly $(X, *, 0)$ is a d-algebra.

Definition 2.3 (Mostafa *et al*., 2017) Let $(X, *, 0)$ be a d- algebra $\emptyset \neq I \subseteq X$ then:

1. I is called a d-sub algebra of X if $x \star y \in I$ whenever $x \in I$ and $y \in I$
2. I is called a d-ideal of X satisfies

ideas brought the hyperstructures to KU-algebras (Sh. Ghorbani and Eslami, 2008). In their discussions of d-algebras as an extension of BCK-algebra, Neggers et al. (1999) and Mostafa et al. (2017) explored many connections between d-algebras and BCK-algebras, as well as oriented digraphs. Fuchsia dot d-sub algebras and fuzzy

D_0. $0 \in I$

D_1. $x * y \in 1 and\ y \in 1 implyx \in 1$

D_2. $x \in 1 and\ y \in X\ implyx * y \in 1$ (1)(2)

(3) $x * y = 0 and\ y * x = 0 Implythat\ x = y, for\ x, yin\ X$(3)

dot d-algebra d-ideals. It was covered that the product of fuzzy dot d-ideals and strong fuzzy relations yields the equivalent strong fuzzy dot d-ideal. Fuzzy dot hyper K-subalgebras, fuzzy dot hyper Definition 2.4 (Jun 1999), and other related topics were covered by Neggers et al. (1999). The following axioms are satisfied by a hyper BCK-algebra with the binary operation o and the constant 0.
K-perfect with additional characteristics.
We provide an overview of Hyper d-Algebra and its associated ideas and features in this research.

finition 2.1 (Neggers *et al*., 1999) A d-algebra is a non- empty setmethods introduced in d-algebra and hyper BCK-algebras.

De X with a constant 0 and a binary operation $*$ satisfying

$x = y$ *for* $x, y \in H$

Results

*Hyper d-Algebras**Definition 3.1.1. Let H be a non-empty set and* $o: H \times H \rightarrow P(H) \backslash \{\phi\}$ *be a hyperoperation. Then (H, o, 0) is called a Hyper d-Algebra if it satisfies the following axioms:* ***Table 2:*** *Hyper d-Algebra*

o	0	a	b
0	{0}	{0, a}	{0, b}
a	{a}	{0, a}	{a, b}
b	{b}	{0, b}	{0, a, b}

We need to show that $A = \{0\}$

$x << x$

$0 << x$

If $x << y$ *and* $y << x$, then $x = y$ for all:(1)

(2)Let for all $x \in A$ there exist $y = 0 \in B$ such that:

$x << y$ *imply* $x << y = 0$ we get $x = 0$

Implies $x \in A$. Hence $A = \{0\}$.

$x, y \in H$(3)Suppose that H is a hyper d-algebra, for all $x \in H$ and

Remark 3.1.2. Let H be a Hyper d-Algebra. Then the following properties hold.

$x \circ y$ can be written as:A, $B \subseteq H$ *and* $A \subseteq B$:

We need to showthat $A << B$

For all $x \in A$. Then there exists $x \in B$ such that $x \ll x$.

$\circ\{y\}, \{x\} \circ \{y\}, \{x\} \circ y$

If $\varphi \neq A, B \subseteq H$, then $A \circ B = \bigcup$ $\circ$ (1)

$_{(a,b) \in A \times B}(a \quad b)$, thenThus $A << B$.

But the converse is not true.

Suppose that H is a hyper d- algebra, for all $x \in H$. We

$A << B$ means for all $a \in A$ there exists $b \in B$ such that: need to show that $\{0\} \subseteq (xox)ox$:

$a << b$(2)$0 \in xox$(1)

$P(H)$ *denotethesetof all subsetsof* H (3) For $x, y \in H$ $x << y$ means $0 \in x$

Example 3.1.3. Let $H = \{0, a, b\}$ be a set. Then the hyperoperation "o" on H is defined by Table 2.

By simple manipulation $(H, o, 0)$ is a hyper d-Algebra. Proposition 3.1.4. Let H be a Hyper d-Algebra. Then for

$\{0\} = 0ox(xox)ox$

$\{0\} \subseteq (xox)ox$

Thus, $0 \in (xox)ox$.

Sub-Algebra of Hyper d-Algebras(2)

(3)

all $x \in H$ and $A, B \subseteq H$. Then each of the following conditions holds:

Definition 4.1.1. Let *(H,* o, 0*)* be a Hyper d-Algebra and*S* be a non-empty subset of *H* containing 0 with respect to

$0o0=\{0\}$

$A << \{0\} implies\ A=\{0\}$

$A \subseteq B implies\ A << B$

$\{0\} \subseteq xo(x\,ox)$

(1)

(2)

(3)

(4)the hyper operation "o" which implies that xoy $\ll S$, for all $x, y \in S$. Then S is called a hyper sub-algebra of H.

Example 4.1.2. Let $H=\{0,1,2,3\}$ be a set. Then the hyperoperation on H is defined by Table 3. Clearly (H, o, 0) is a Hyper d-Algebra.

Let $S=\{0,1,2\}$ be a subset of a Hyper d-Algebra (H, o, 0). Since:

Proof 1. Let H be a Hyper d-Algebra. Then $x << x$, for all $x \in H$.

We want to show that $0o0=\{0\}$ Since:$1o2=\{1,2\} << S, 1 \in S\ and\ 2 \in S$

$2o2=\{0,2\} << S=\{0,1,2\}, 0 \in S\ and\ 2 \in S$

$1o1=\{1\} << S=\{0,1,2\}, 1 \in S$(1)

(2)

(3)

$x << x \Rightarrow 0 \in x \circ x$

If we put $x = 0$ in the form $x << x$ we get $0 \circ 0=\{0\}$

$Thus, 0o0=0\}$ .1)Thus, S is a sub-algebra of a Hyper d-Algebra.

Proposition 4.1.3. Let S be a non-empty subset of a Hyper d-Algebra $(H, \mathrm{o}, 0)$ and $x\mathrm{o}x \ll S$, for all $x \in S$. Then $0 \in S$.
Proof. Let S be a non-empty subset of a Hyper d-Algebra

Table 3: Hyper d-subalgebra

o	0	1	2	3
0	{0}	{0,1}	{0,2}	{0,3}
1	{1}	{0,1}	{1,2}	{1,3}
2	{2}	{0,2}	{0,2}	{0,2,3}
3	{3}	{1,3}	{2,3}	{0,1,2,3}

Theorem 4.4. Let $(H, \mathrm{o}, 0)$ be a Hyper d-Algebra and $S=\{x \in H \mid 0\ o\ x \ll \{0\}\}$. Then S is a Hyper d-subalgebra of H. Proof. Let $(H, o, 0)$ be a hyper d-algebra and $S=\{x \in$

Table 4: Hyper d-IDEAL of H

0	0	a	b	c	d
0	{0}	{0, 0, a}	{0, 0, b}	{0, 0, c}	{0, 0, 0, 0, c}
a	{a}	{0, a, b}	{0, a, b}	{0, b, c}	{0, 0, 0, b, c}
b	{b}	{0, 0, b}	{0, 0, b}	{0, b, c}	{0, 0,0, 0, a}
c	{c}	{0, a, c}	{0, c, a}	{0, 0, c}	{0, 0, 0, c, a}
d	{d}	{0, a, b}	{0, b, c}	{0, a, b}	{0, a, b, c, d}

Table 5: Hyper d-Ideal

| $0\ o\ x \ll \{0\}\}$ and let $x, y \in$ S. Then $x = 0\ o\ x \ll \{0\}$ and $y = 0\ o\ y \ll \{0\}$.

Now $xoy = (0ox)o(0oy) \ll \{0\}o\{0\} = \{0\}$, Imply that $xoy \ll \{0\} \Rightarrow xoy \ll S$. Thus, S is a hyper d-subalgebra of H.

0	0	a	b	c
0	{0}	{0, 0, a}	{0, b}	{0, b}
a	{a}	{0, 0, a,}	{a, b}	{0, b}
b	{b}	{0, 0, b}	{0, b}	{0, b}
c	{c}	{0, a, b}	{0, a}	{0, 0}

Hyper d-Ideals of Hyper d-Algebras

Definition 5.1.1. Let I be a non-empty subset of a Hyper d-Algebra H. Then I is called a Hyper d-Ideals of H if it satisfies the following axioms:Proof. Assume I am a Hyper d-Ideal of a Hyper d-Algebra H.

Since $I \neq \emptyset$, for all $x \in I, x \ll x$, we have $0 \in xox \subseteq I$.
Thus, $0 \in I$.

$I_0.\ 0 \in I$

$I_1.\ x \circ y << I$ and $x \in I$ imply $y \in I$

$I_2.\ x \in I$ and $y \in H$ imply $x \circ y << I$, for all $x,y \in H$(1)

(2)

(3)Proposition 5.1.6. Let I be a Hyper d-Ideal of a Hyper d-Algebra H. If $y \ll x$ and $x \in I$, then $y \in I$.

Proof. Let I be a Hyper d-Ideal of a Hyper d-Algebra H such that $y \ll x$ and $x \in I$.

Since $y \ll x \Rightarrow 0 \in y\ o\ x \subseteq$I $\Rightarrow 0 \in I$. (proposition 5.1.3.)

Example 5.1.2. Let $H = \{0, a, b, c, d\}$ be a set. Then the hyperoperation "o" on H is defined by Table 4.

With simple calculation $(H, o, 0)$ is a Hyper d-Algebra.

Let $I = \{0, a, b, c\}$ be a subset of H. Then I is a Hyper d-Ideal of H. Since:Consequently, $yox \ll I$ and $x \in I$ imply $y \in I$.
Therefore, $y \in I$.

Definition 5.1.7. Let H be a Hyper d-Algebra. Then a hyper d-ideal I of H is called a Hyper d$^{\#}$-ideal of H. $xoz \ll I$ whenever $xoy \ll I$ and $yoz \ll I$, for arbitrary $x, y, z \in H$.

$I_0.\ 0 \in I$

$I_1. a \circ b << I$ and $a \in I$ implyb $\in I$ $I_2.\ c \in I$ and $d \in H$ implyc

Thus, I is a Hyper-d-Ideal of H.(1) $\circ d << I$

(2)

(3)Example 5.1.8. Let $H = \{0, a, b, c\}$ be a set. Then the hyperoperation "o" on H is defined by Table 6.

Clearly $(H, o, 0)$ is a hyper d- Algebra. Let $I = \{0, a, b\}$ be a subset of a hyper d-Algebra. Then:

$\circ$ $\circ$ (1)

Proposition 5.1.3. Let I be a Hyper d-Ideal of H and let A be a subset of a Hyper d-Algebra H such that $A \ll I$. Then $A \subseteq I$.

Proof. Let I be a Hyper d-Ideal of H and let A be a subset of H such that $A \ll I$. Then for all $a \in A$ there exist $x \in I$ such that $a \ll x \Rightarrow 0 \in aox \ll I$. $x \in I$ imply $a \in I$, (Since I is a Hyper d-Ideal of H). Thus, $A \subseteq I$.

$a \circ c = \{0,b,a\} << I$, whenever $a \circ a = \{0,a\} << I$

and $a \circ c = \{0,b,a\} << I$

$a \circ 0 = \{a\} << I$, whenever $a \circ b = \{a,b\} << I$

and $b \circ c = \{b\} << I$

Example 5.1.4. Let $H = \{0, a, b, c\}$ be a set. Then the hyperoperation "o" on H is defined by Table 5
Clearly $(H, o, 0)$ is a Hyper d-Algebra.

$a \circ c = \{0,b,a\} << I$, whenever $a \circ a = \{0,a\} << I$

and $a \circ c = \}0,b,a\} << I$

Let $I = \{0, a, b\}$ be a non-empty subset of a Hyper d-Algebra H. Then:

(2)
(3)
(4)

$I_0.\ 0 \in I$

$I_1. a \circ b = \{a,b\} << I = \{0,a,b\}\ and\ a \in I \quad imply \quad b \in I$

$I_2.\ a \in I\ \ and\ \ c \in Himplya \circ c << I$

Thus, *I* is a Hyper-d-Ideals of a Hyper-d-Algebra.

(1)

(2) $a \circ b = \{a,b\} << I,\ whenever\ a \circ c = \{0,b,a\} << I$
$and\ \ c \circ b = \{0,a\} << I$

(3)
$Thus, I$ is a hyper $d^{\#}-$ ideal of H.

Definition 5.1.9. Let I be a Hyper d$^{\#}$-Ideal of a Hyper d-Algebra H satisfies $xoy \ll I$ and $yox \ll I$ imply $(xoz)o(yoz) \ll I$ and $(zox)o(zoy) \ll$ I, for all $x, y \in H$. Then I is called a Hyper d*-Ideal of H.

Example 5.1.10. Let $H = \{0, 1, 2, 3, 4\}$ be a set. Then the

Lemma 5.1.5. If I is a Hyper d-Ideal of a Hyper d-Algebra H, then $0 \in I$.hyperoperation "o" on H is defined by Table 7.

Clearly $(H, o, 0)$ is a Hyper d-Algebra.

o	0	a	b	c
0	{0}	{0, a}	{0, b}	{0, 0, a}
a	{a}	{0, a}	{a, b}	{0, b, a}
b	{b}	{0, b}	{0, b}	{0, 0, b}
c	{c}	{0, a}	{0, a}	{0, a, c}

Table 7: Hyper d*-ideal

o	0	1	2	3	4
0	{0}	{0, 0, 1}	{0, 2}	{0, 2}	{0, 0, 0}
1	{1}	{0, 0, 1}	{1, 2}	{0, 0}	{0, 0, 1}
2	{2}	{0, 1, 2}	{0, 2}	{1, 2}	{0, 0, 2}
3	{3}	{0, 1, 2}	{0, 1}	{0, 1}	{0, 0, 2}
4	{4}	{0, 0, 2}	{0, 1}	{0, 1}	{0, 1, 2}

Let I= $\{0,1,2,3\}$ be a subset of a Hyper d- Algebra.

Since, $0o1 = \{0,1\} \ll I$ and $1o0 = \{0,1\} \ll I$ imply $(0o2)o(1o2) = \{0,1,2\} \ll I = \{0,1,2,3\}$ and $(2o0)o(2o1) \ll I = \{0,1,2\} \ll I = \{0,1,2,3\}$ and $3o4 = \{0,2\} \ll I$ and $4o3 = \{0,1\} \ll I$ imply $(3o3)o(4o3) = \{0,1\} \ll I = \{0,1,2,3\}$ and $(3o3)o(3o4) \ll I = \{0,1,2\} \ll I = \{0,1,2,3\}$. Thus, I is a Hyper d*-Ideal of H.

Discussion

Following the release of hyper BCK-algebra in 1999, other varieties of hyper algebraic structures appeared. In light of these results, we investigate novel concepts of Hyper d-Algebra, Hyper d-Ideals, and Hyper d*-Algebra that exhibit the properties listed in the main result. The primary methodologies used in this investigation were direct and indirect evidence as well as proof by contradiction.

Conclusion

This study delves into hyper d-algebra, a new concept with several definitions. It is possible to extend this idea to other algebraic structures by giving it more properties.

Reference

In the year 2020, Dejen was published. Construction of Fuzzy Dot d-subalgebras. Volume 4, Issue 4, pages 130–136, Journal of Applied Mathematics and Computation. This article is cited as https://doi.org/10.26855/jamc.2020.12.00400.

[D. T. Gerima] (2022). In terms of the fuzzy dot hyper K-ideal of a hyper K-algebra. 23(3), 285–294 theo. AFMI-H-220121R2.pdf may be found at http://www.afmi.or.kr/articles_in_%20press/2022- 03/AFMI-H-220121R2. Yousif Bin Jun (1999). Scalar elements and hyperatoms of hyper BCK-algebras. Volume 2, Issue 3, pages 301-309.

citation: "ions container" from tps://cir.nii.ac.jp/crid/1571417125840854912. Yousif Bin Jun (2001). Hyper BCK algebras that are sitive and implicative. Online Journal of Mathematical Sciences in Japan, 5, 67-76. This information is from Jun and Zahedi (2000). Concerning Hyper Black-Algebras. Published in the Italian Journal of Pure and Applied Mathematics, volume 10, pages 127–136. Hello there! 636719/en https://sid.ir/paper/ Written by Mostafa, S. M., Bijan, D., and Fatema, F., K. in 2017. Hyper structure theory on KU-algebras used. Article cited as Journal of Hyperstructures, 6(2), pages 82–95. This is the link to the article tps://jhs.uma.ac.ir/article_2663_9a5c1f1f7e7d077 7983c7ef5ac60064a.pd.

In 1999, Neggers, Jun, and Kim published a paper. Concerning d-ideals found in d-algebras. Journal of Slovaca Mathematica, 49(3), 243251. MathSlov_49-1999-3_2.pdf may be found at this

URL: https://dml.cz/bitstream/handle/10338.dmlcz/129056. In 2014, Radfar, Rezaei, and Saeid published a paper. Highly algebraic sets. The article is published in the Novi Sad Journal of Mathematics, volume 44, issue 2, articles 137–147. The requested URL is not a valid URL; ase provide the full URL.Authors: Sh. Ghorbani, A. H., and E. Eslami (2008). Hyper MV-algebras. Publications in Set-Valued Mathematics, 2, 205-222. With M, M. (2006) as the author. An article by Zahedi titled "Italian Journal of Pure and Applied Mathematics, 20, 121-138" discusses commutative hyper K-ideals and quasi-commutatives.