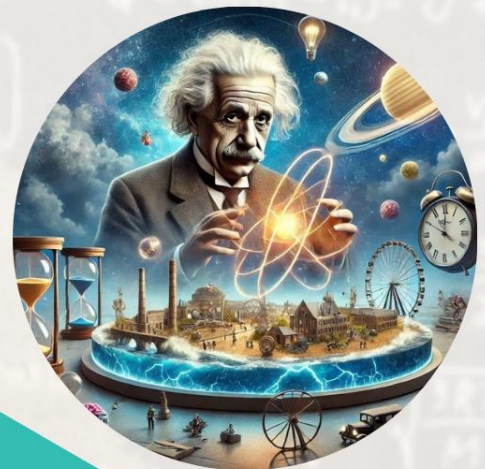


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# Pricing Normal Bonds in a Stochastic Model with Cross-Over Maturity to a Pandemic Period

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## Article Info

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**Abstract:** The current research presents an Ito form for conventional bond trading in which the maturity periods overlap with the COVID-19 pandemic era. Path reversals corresponding to their canonical routes are shown to occur in normal bonds during this time. Additionally, the criteria that were used to get this astonishing outcome are laid forth. One important thing that bondholders should do is pass regulations that allow for variable pricing. This will help the issuer stay afloat during the current COVID-19 epidemic by following the opposite route that this research found.

**Keywords:** CAT Bond, Sub-Exponential Shock, Noise, Wiener Process, Issuer

## Introduction

The emergence of the COVID-19 pandemic has changed the course of several random processes. With 6,844,645 fatalities reported as of February 6, 2023, the total number of affected individuals is 671,706, 853. A reasonable and probable assumption is that international bond dealers constitute a significant portion of these numbers. According to Liang (2020), the bond market suffers massive losses as a result of quick sales at giveaway prices if a large portion of bond dealers are infected with COVID-19. Therefore, without proper trading measures, economic collapse and meltdown may occur in nations renowned for bond raising if there are several COVID-19 infected bond dealers. To prevent growing pandemic arbitrage, new hybrid bond pricing models should be used. If you would want to learn more about hybrid models and how they may be used for optimization, we suggest reading (Alhawarat et al., 2021; Salleh et al., 2022).

Holders will also have a hard time getting their fair share of coupon payments if a large number of issuers die quickly, as happened with the feared COVID-19 pandemic data mentioned before. Bond companies would encounter major problems including complete collapse and resource loss if this happens, and they won't be able to pay their secured and unsecured debts (Brown, 2006). An awful case in point is the demise of Silicon Valley Bank in the United States. When nations who are issuing bonds are implementing COVID-19 limitations, such as lockdowns, it is very probable that bondholders will not get payment from issuers. Due to changes in employment, issuers are not receiving their full salary and pay in this instance.

COVID-19 pandemic period, which will help investors save money. The announcement for investment grade bonds, for example, was made on 23rd March 2020. There was no discernible impact on stock prices, although the news did drive up bond prices by 7% upon exercise. The unexpected correlation between bond and stock prices scared away bond investors who were worried about the increased risks associated with bond dynamics in the wake of the pandemic (Haddad et al.,

2021). The current state of the global bond market is precarious, with rising default rates and increasing risks (Zaghini, 2023). The importance of considering the following real-world scenarios: feedback effect (Sani et al., 2020; Edmans et al., 2015; Cofnas, 2016), ripple effect (George and Beard, 2023), or maturity periods taking longer than expected default times

to wade away price informativeness (Dávila and Parlatore, 2018) due to the COVID-19 pandemic. Here, mispricing occurs in the short and long runs because the feedback effect alters bond speculators' decisions. For example, bond markets will see a decline in their profit margins relative to stock markets over the long run. This is because stock market profit margins are dependent on the accuracy of time and space pricing models (Joyner, 2006). As a result, generating updated bond pricing models is critical, particularly in cases when the maturity dates coincide with the pandemic period. We build a novel cross-over bond stochastic pricing model with this in mind. The maturation period begins with

redefined in view of lengthy default tendencies (Goldie and Kluppelberg, 1988; Mikosch, 1999; Foss et al., 2013).

Chao and Zou (2018) studied flood related Catastrophe (CAT) bond prices and established the CIR-copula-POT model with stochastic rates showing that maturity dates affect bond prices. Shao (2015) considered both seismic and nuclear catastrophes and showed that bond prices are

We consider a bond whose non-pandemic trading time price at maturity  $p(\cdot, T)$  is quoted as:

$$p(\cdot, T) = \mathbb{E} \left[ \frac{\phi}{(1+\chi)^T} \right] \quad (1)$$

affected by extended maturities and defaults (Shao et al. (2017) developed a semi-Markov model to analyze the claim's inter-arrival periods by viewing CAT bond prices as a series of compound inhomogeneous Poisson processes perturbed by a diffusion process. Based on the results of the numerical study, CAT bond prices will fall when the default probability, time to maturity, and threshold level are all raised. In their study of the relationship between CAT bonds and the financial market during an era of increasing default risks, Lee and Yu (2002) shown that moral hazards and baseline risks may substantially reduce bond trading.

Nowak and Romaniuk (2017) studied CAT bonds related

where,  $T \geq 0$  is the time to maturity parameter of  $\cdot$ ,  $\varepsilon$  is the

ratio of monthly coupon payment to prevailing monthly market interest rate,  $\chi$  is one on prevailing monthly market

interest rate and  $\phi$  is the face value of  $\cdot$ . We suppose that the same  $\cdot$  is to be traded and exercised in the present COVID-19

pandemic period with  $p(\cdot, T) \rightarrow p(\cdot, F(T))$  such that:

$$p(\cdot, F(T)) = \mathbb{E} \left[ \frac{\phi}{(1+\chi)^{F(T)}} \right] \quad (2)$$

to investor risk-taking and showed that adjusted bond pricing models can wade away arbitrage tendencies better than non-adjusted bond pricing models (Romaniuk, 2003). Ma and Ma (2013) studied CAT bonds with non-homogeneous Poisson losses given interest rate uncertainty, loss severity, and claim arrival intensities and showed that interest rates rise bond prices when the maturity time bracket is  $[0.25, 2.5]$ . Hofer et al. (2020) studied CAT bond pricing where associated risk is defined in a fixed interval and showed that threshold losses and expiration times have significant roles in arbitrage-free pricing. Tao et al. (2009) studied pricing models for earthquake-related catastrophes and stated the needed techniques for estimating personal insurance under seismic related deductibles. Vaugirard (2004) stated that the subject of pricing CAT bonds boils down to computing the first passage times for some jump diffusion stochastic

Here,  $F \in Z$  is a bridge-in-price function sequel to changing trading intervals generated by the COVID-19 pandemic on  $\cdot$  with the property that:

If the path of  $F(\cdot, T)$  assumes the path of a wiener process  $\{W(\cdot, T): T \geq 0\}$  through the maturity parameter

$T \leq t$ , then the guarantee of the existence of at least one natural homomorphism  $\varphi$  such that:

processes under certain risk assumptions.

$$\mathbb{T} \times \Omega \rightarrow \mathbb{R}$$

$$\lim_{T \rightarrow \infty} \frac{\bar{F}^{n*}(\mathbb{B}, T)}{\bar{F}(\mathbb{B}, T)} = n, n \geq 2 \tag{3}$$

Egami and Young (2008) studied CAT bonds where inter-arrival claims follow the Poisson diffusion process and showed that prices are indifferent. Ma *et al.* (2017) developed a bond pricing model for zero coupon CAT bonds with stochastic Poisson arrivals and showed that hazards and interest rate risks affect zero coupon bond prices. In general, it is imperative to note that CAT bonds are hedging financial options as in Galeotti *et al.* (2013);

is established. In this case,  $\phi$  is a homomorphism between rings with identities onto  $\mathbb{R}$ .

*Proposition 1*

The homomorphism  $\phi$  in (4) such that:

Bodoff and Gan (2009); Lai *et al.* (2014); Gürtler *et al.* (2016). De Spiegeleer and Schoutens (2011); Baz and Chacko (2004); Rebonato (2018) described how existing bond models devoid of extra-uncertainties (Szczygielski *et al.*, 2021; Lyócsa *et al.*, 2020; Albulescu and Grecu 2023) are distinct from those ones where coupon payments assumed the paths of long-tailed processes (Norman *et al.*, 2020). Thus for completeness, linking the times to maturity with the pandemic time is important for aided efficiency especially where default times assumed the path of the COVID-19 pandemic is sub-exponential with parameter  $\gamma \geq n$  provided that  $\phi$

is measurable.

*Proof*

Fix  $(\omega \in \Omega) \rightarrow x$  and apply the isomorphism of (Arnautov and Ermacova, 2014; Kulabokhov, 2019) on the right-hand side of (5). Consequently:

$$\lim_{T \rightarrow \infty} \frac{\bar{F}^{n*}(T, x, W(T, \mathbb{B}))}{\bar{F}(T, x, W(T, \mathbb{B}))} \stackrel{\gamma}{=} \lim_{T \rightarrow W(T, \mathbb{B})} \frac{\bar{F}^{n*}(W(T, \mathbb{B}))}{\bar{F}(W(T, \mathbb{B}))} \tag{6}$$

$$C - \ln \varepsilon \frac{\partial F(T, W(T, \mathbb{B}))}{\partial W(T, \mathbb{B})} \tag{14}$$

The mini parameter  $\gamma \in \mathbb{R}$  is the intensity due to  $x$  fixed. Thus, one can write (6) in view of (3) respective of a natural number  $n$  as: Finally:

$$\lim_{T \rightarrow \infty} \frac{\bar{F}^{n*}(T, x, W(T, \mathbb{B}))}{\bar{F}(T, x, W(T, \mathbb{B}))} \leq \gamma \lim_{T \rightarrow W(T, \mathbb{B})} \frac{\bar{F}^{n*}(W(T, \mathbb{B}))}{\bar{F}(W(T, \mathbb{B}))} \leq n \tag{15}$$

(7) Here again:

$$C - \ln \varepsilon^2$$

$$\frac{\partial^2 F(T, W(T, \mathbb{B}))}{\partial W^2(T, \mathbb{B})} \tag{16}$$

ince,  $n \geq 2$  in (7), the bond price  $p(\bullet, T)$  in (1) must

admit some sub-exponentially due to induced COVID-19 pandemic effects as in (2). In this case, the quoted price

$p(\bullet, \bar{F}(T))$  in (2) holds good whenever  $\bullet$  is within the dreaded<sup>2</sup> pandemic period with the representation<sup>3</sup> as in (7) and such that:

And that:



$\sigma W(T, (21)_0)$   
 Here:

$$^2 \varepsilon = \frac{C_m}{r}$$

$$^3 \chi = 1 + r_m$$

$$= p \exp\left(\alpha - \frac{1}{2} \sigma^2 T\right)$$

$$\alpha = \left[ C_0 + \frac{1}{2}(C_2 + C_3) \right]$$

**Table 1:**  $\alpha, \sigma$  Trends versus T in COVID-19 Times

T(years)	$\alpha$	$\sigma$
0	11.51300000	0.0000000
1	21.34200000	3.4112000
2	5.92190000	0.8528100
3	1.62960000	0.2076000
4	0.64778000	0.0631710
5	0.33685000	0.0233970
10	0.05464200	0.0008679
35	0.00181910	1.7451e-06
50	0.00065513	2.9402e-07
70	0.00024677	5.4734e-08
85	0.00013987	2.0741e-08
100	8.67930e-05	9.2048e-09

Table 1 shows  $\alpha$  and  $\sigma$  paths for selected  $T$ . From the said table, the impact of the noise parameters can be deduced. For instance, in 2023, bond prices will drift about 65% of their non-pandemic face values at 6% frequency due to the COVID-19 pandemic. Interestingly in 2029, the price of the same asset drifts only around 5% at a frequency lesser than 1%.

## Results and Discussion

In order to examine the trajectory of (2) through the lens of the 80-20 Pareto distribution, we run simulations of (21) with the assumption that 20% of bondholders are infected with the COVID-19 pandemic, and 80% of the bond wealth is held by this 20%. In addition, we examine the changes in the route between the two models by simulating the bond price model in (1). The points that follow are valid. Comment 1 In the event where T is Pareto 80-20 and grows positively in R as a result of defaults, then  $C_0, C_1,$  and  $C_3$  will all drop, but  $C_2$  will rise. The constant values  $C_0, C_1, C_2,$  and  $C_3$  are shown versus the time interval  $T \leq 100$  years in Table 2.  $C_0$  values gradually drop to 0.05 at  $T = 10$  years, as shown in the aforementioned table. A pattern of  $C_0 \approx 0$  remains stable for  $T > 10$  years after this trend. Consistent with a bond constant that decreases with increasing T, the overall trend for  $C_0$  is clear. Here,  $C_0$  stands for the part of the interest rate that was prevalent throughout the epidemic. Once again, at  $T = 5$  years,  $C_1$  drops to 0.02 according to Table 2.  $C_1 \leq 0$  thereafter for all  $T > 5$  years follows this pattern. During the pandemic phase,  $C_1$  tends to decrease as T rises. As T changes,  $C_3$  follows the same pattern in its trajectory.  $C_2$ , in contrast, begins at 3.4112 at  $T = 1$  year and rises gradually to 0.04 at  $T = 4$  years. It is reasonable to assume that the COVID-19 pandemic's concealed impacts on the current model (1) are the source of the declining  $C_i$ 's.

### Remark 2

Suppose  $B$  is quoted where  $T$  is sub-exponentially 80-20 Pareto-tailed sequel to the COVID-19 pandemic or any of its arguments. Then  $p(\cdot, F(T))$  is astronomical in the short term.

Table 3 presents the path analysis of  $p(\cdot, F(T))$  as in (18) and that of  $p$  as in (1). Clearly, at  $T = 1$  year,  $p_T$  skyrockets from \$1000 to an astronomical value of \$16,729,000,000. Again, At  $T = 2$  years and  $T = 3$  years respectively, the bond price values are \$370,310,000 and \$232,080. Here, the COVID-19 pandemic created 80% uncertainties in the bond market causing investors to flee to other financial markets as in Ma and Ma (2013) under high-level uncertainty, trader loss severity, and high claim arrival intensities. Bond traders will choose to cash in to treat COVID-19 pandemic symptoms without minding the price. As a result, bond markets lose control over the short-term

maturities. Afterward, bond prices show signs of stability in both time and space. For instance, in the year 2029, the bond exercised at \$1000 will trade at \$1494 via the canonical model and at \$1742 via the stochastic model developed in this study.

**Remark 3**

Under the 80-20 pareto COVID-19 pandemic tail, the price of  $\bullet$  reverses its path to a monotonically decreasing path within the same maturity interval.

This information is clear from Table 3 showing that the price of the bond  $p$  climbs consistently from \$1000 at  $T=0$  to approximately \$6000 at  $T=100$  years. On the other hand,  $p(\bullet)$ ,

$F(T)$  declines towards extremely low values close to \$1000 as time to maturity  $T$  approaches 100 years. In this case, the paths assumed by the two models are opposite in direction and strictly monotonic agreeing with (Shao *et al.*, 2015) that showed that the price of CAT bonds is most likely to reverse when the threshold level is decreased, the time to maturity is increased and the likelihood of default increases. This further proves the strength of the methodology designed and presented in this study.

**Table 2: C\_0, C\_1, C\_2, C\_3 versus T in COVID-19 times**

T(years)	C_0	C_1	C_2	C_3
0	11.5130000	0.0000000	0.0000000	0.0000000
1	3.41120000	3.4112000	-3.4112000	39.273000
2	1.43910000	0.8528100	-0.8528100	9.8183000
3	0.73683000	0.2076000	-0.1698500	1.9555000
4	0.42640000	0.0631710	-0.0421140	0.4848600
5	0.26852000	0.0233970	-0.0129980	0.1496500
10	0.05330100	8.6791e-04	-2.5530e-04	2.9389e-03
35	0.00181830	1.7451e-06	-1.4933e-07	1.7193e-06
50	0.00065504	2.9402e-07	-1.7627e-08	2.0294e-07
70	0.00024676	5.4734e-08	-2.3448e-09	2.6995e-08
85	0.00013987	2.0741e-08	-7.3182e-10	8.4254e-09
100	8.67910e-05	9.2048e-09	-2.7609e-10	3.1786e-09

**Table 3: p and p\_T versus T in COVID-19 times**

T(years)	P(\$)	P_T(\$)
0	1000.0	1000.0000000
1	1049.5	1.6729e11
2	1098.9	3.7031e08
3	1148.4	2.3208e05
4	1197.8	17044.0000000
5	1247.1	6048.8000000
10	1493.6	1742.1000000
35	2717.0	1065.8000000
50	3443.7	1033.3000000
70	4404.2	1017.4000000
85	5118.3	1012.0000000
100	5827.1	1008.7000000

T(years)	P(\$)	P_T(\$)
0.0	1000.0	1000.000000
1.0	1049.5	1.6729e11
2.0	1098.9	3.7031e08
3.0	1148.4	2.3208e05
4.0	1197.8	17044.000000
5.0	1247.1	6048.800000
10.0	1493.6	1742.100000
11.0	1542.9	1608.900000
11.1	1547.8	1597.800000
11.2	1552.7	1587.000000
11.3	1557.6	1576.500000
11.4	1562.6	1566.300000
11.5	1567.5	1556.300000
11.6	1572.4	1546.700000
11.7	1577.3	1537.300000
11.8	1582.2	1528.200000
11.9	1587.2	1519.300000
12	1592.1	1510.700000
13	1641.3	1435.600000
35	2717.0	1065.800000

*Remark 4*

There exists a unique price for B identical to both models. Table 4 shows  $p(\bullet, F(T))$  when  $T = 11.2$  years is \$1587 which coincides with  $p$  at  $T = 11.9$  years. At this point, the coincidental price represents the equilibrium price where the canonical model perfectly transformed itself onto the stochastic model. By proposition 1,  $p(\bullet, F(T))$  extends the canonical model onto the COVID-19 pandemic times at this maturity period. As a consequence, it is clear that the stochastic model has a root in the canonical model as claimed and analyzed in this study.

*Remark 5*

The 80-20 pareto tail containing COVID-19 trading times are normal bonds under long  $T$ .

Table 3 shows  $p(\bullet, F(T))$  against selected values of  $T$ . From the said table, it is clear that a bond with a \$1000 face value has  $p(\bullet, F(T))$  ranging from \$1742.1 to \$1008.7

for  $T \geq 10$ . Clearly here, the pattern of long maturity times is being much closer to the face value of the bond during the COVID-19 pandemic. Again, bond prices enter the normal region in volatility hence, normal under maturity periods  $T \geq 10$ .

**Conclusion**

We offer an Ito representation and analysis of a CAT bond pricing model that we developed for regular bond trading during the COVID-19 epidemic. Findings from this study corroborate those from Romaniuk (2003), which found a correlation between the COVID-19 pandemic and seismic CAT bonds. Once again, we have shown that COVID-19 pandemic CAT linkages may undergo route reversals. For CAT bonds affected by earthquakes, this finding is in

agreement with (Shao et al., 2015). It is recommended to issue bonds with mid-term maturities in order to mitigate anticipated risks, since short-term CAT bonds become prohibitively costly in the 80-20 pareto tail area that includes the COVID-19 pandemic. Lastly, the study suggests that between the eighth and twelfth years of the COVID-19 pandemic, CAT bonds inside the 80-20 pareto tail that includes COVID-19 trading times stabilized across lengthy maturity periods.

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